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High intensity Compton scattering off particles with anomalous magnetic moment

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Abstract. For a particle with an anomalous magnetic moment the differential cross section for the emission of a photon due to interaction with an intense external wave field of circular polarization is calculated. The scattered frequencies appear in triplets of very small separation and there is in addition a 'zeroth' harmonic of very low frequency. In the cross section the contributions from the anomalous moment add to the relativistic terms in the corresponding formula for normal particles. The transition probabilities for all three numbers of the triplets differ from each other. For neutrons only the zeroth, first and second harmonics can appear. The intensity of the zeroth harmonic turns out to be very low both for electrons and nucleons.

1. Introduction

The interaction of charged particles with an intense wave field, which may be produced by a laser, differs in some aspects from the well known Compton interaction. The Compton formula for the frequency of the emitted quantum is modified by an additional, intensity-dependent shift and by the appearance of higher harmonics of the laser frequency (Brown and Kibble 1964). If the particle has in addition to its Dirac moment an anomalous magnetic moment and the laser field is circularly polarized, there should also appear a scattered quantum of very low (ie radio) frequency (Becker 1975). This 'zeroth-order harmonic' is entirely due to the interaction of the anomalous moment with the laser. A detection of this frequency would control the theoretical description used for the interaction of the particle with the laser. In general, one would expect the intensity of the zeroth-order harmonic term to be small: magnetic interactions are relativistic effects, which are hard to measure in Compton scattering, if the incident photon energy is small compared to the rest energy of the particle. Since it is, however, possible to detect very weak radio signals, one needs more precise statements on the magnitude of the effect. We shall therefore calculate in this paper the cross section for high intensity Compton scattering off particles with an anomalous magnetic moment and shall investigate the quantitative aspects in some detail. The calculation is based on the wavefunction as given in an earlier paper (Becker and Mitter 1974, to be referred to as I) and generalizes the corresponding derivation for 'normal' particles (Brown and Kibble 1964).

2. The matrix element

Let the wavefunctions for the particle in the initial (final) state be characterized by the momentum p (p') and the outgoing photon have the momentum q and polarization ϵ'. Then the matrix element is

$$\langle p'q\epsilon'|p\rangle = -\mathrm{i}e\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}(qx)}\bar{\psi}_{\mathrm{f}}(x|p')\left(\gamma_\mu\epsilon'^\mu+\frac{\mathrm{i}\mu}{2m}q^\mu\epsilon'^\nu\sigma_{\mu\nu}\right)\psi_{\mathrm{i}}(x|p). \tag{1}$$

Here μ is the anomalous magnetic moment. The wavefunction ψ has been computed in I. If the result I (24) is inserted and the integration is performed in light-like co-ordinates, we obtain

$$\langle p'q\epsilon'|p\rangle = -\mathrm{i}e(2\pi)^3\,\delta(p'_v+q_v-p_v)\,\delta(p'_i+q_i-p_i)\,\mathrm{e}^{\mathrm{i}\phi_0}\int \mathrm{d}u\,\mathrm{e}^{\mathrm{i}\phi}(\bar{\psi}_{0\mathrm{f}}T\psi_{0\mathrm{i}}). \tag{2}$$

Here ϕ_0 is a constant phase and we have†

$$\phi = uq_u+\frac{1}{2p'_v}\int^u \mathrm{d}\bar{u}(m^2+\pi'_i\pi'_i)-\frac{1}{2p_v}\int^u \mathrm{d}\bar{u}(m^2+\pi_i\pi_i) \tag{3}$$

$$T = (c_1-\mathrm{i}\sigma_3c_2+\gamma_ib_i)\left(\mathbb{P}_u+\frac{1}{2p'_v}(\gamma_i\pi'_i+m)\gamma_v\right)\left(\gamma_\mu\epsilon'^\mu+\frac{\mathrm{i}\mu}{2m}q^\mu\epsilon'^\nu\sigma_{\mu\nu}\right)\left(\mathbb{P}_v+\gamma_v\frac{1}{2p_v}(\gamma_i\pi_i+m)\right)$$
$$\times(c_1+\mathrm{i}\sigma_3c_2+\gamma_ib_i). \tag{4}$$

The evaluation of the latter expression is straightforward, but yields a rather lengthy formula. The matrix element can be shown to yield results which are invariant under a gauge transformation

$$\epsilon'^\mu \to \epsilon^\mu = \epsilon'^\mu+\alpha q^\mu. \tag{5}$$

Most of the expressions obtained from equation (4) are explicitly invariant under this substitution because of $q_\mu\epsilon'^\mu = q_\mu q^\mu = 0$. In those which are not invariant, the transformation produces an additional term, which integrates to zero. We can therefore use a convenient gauge. Choosing

$$\alpha = -\epsilon'_v/q_v \tag{6}$$

the expression for T simplifies considerably and takes the form

$$T = \frac{mq_v}{2p_vp'_v}(D+\mathrm{i}E\sigma_3\gamma_v+F_j\gamma_j\gamma_v) \tag{7}$$

where we have

$$D = K\frac{p_v+p'_v}{q_v}\epsilon_iR_i \tag{8a}$$

$$E = (b_ib_i-c_ic_i)N-2(c_2b_j+c_1b_i\epsilon_{ji})L_j \tag{8b}$$

$$F_j = (c_1^2-c_2^2-b_ib_i)L_j+2(c_1c_2\epsilon_{kj}+b_kb_j)L_k+2(c_1\epsilon_{ji}b_i+c_2b_j)N \tag{8c}$$

† We shall use the notation of I, except that we shall put $\hbar = c = 1$.

with

$$L_j = (1+\tfrac{1}{2}\mu)\epsilon_j + \tfrac{1}{2}\mu\epsilon_k(2R_kR_j - \delta_{kj}R_lR_l) \tag{9a}$$

$$N = (1+\mu)\epsilon_k\epsilon_{kl}R_l \tag{9b}$$

$$K = c_ic_i + b_ib_i = \text{constant} \tag{9c}$$

and the two-vector

$$R_i = \frac{p_vp'_v}{mq_v}\left(\frac{p_i}{p_v} - \frac{p'_i}{p'_v}\right) + \frac{ea}{m}a_i(u). \tag{10}$$

For later use we note, that R_i may be written in the form

$$R_i = -\frac{1}{m}\left(\pi_i - \frac{q_i}{q_v}\pi_v\right) \tag{11}$$

since a_v is zero.

3. Spin and polarization sums

In order to calculate the averaged cross section we have to evaluate the expression

$$Z(u,u') = \tfrac{1}{2}\sum_{\text{spin}}\sum_{\text{pol}}(\bar{\psi}_{0\text{i}}T^\dagger(u')\psi_{0\text{f}})(\bar{\psi}_{0\text{f}}T(u)\psi_{0\text{i}}). \tag{12}$$

The spin sum can be computed by the trace method. Using the conventional Pauli spinors for ψ_0 we obtain

$$Z(u,u') = \tfrac{1}{8}\sum_{\text{pol}} \mathrm{Tr}\, T^\dagger(u')(1+\gamma^0)T(u)(1+\gamma^0) = \frac{m^2q_v^2}{8p_v^2p_v'^2}\sum_{\text{pol}}(D'^*D + E'^*E + F_j'^*F_j)$$

where the prime refers to the argument u'. At this stage it is very convenient to introduce the combinations (cf Becker 1975)

$$\begin{aligned} S_1 &= b_ib'_i + c_ic'_i, \qquad S_3 = \epsilon_{ij}(b_ib'_j - c_ic'_j) \\ S_{2j} &= c_1b'_j - c'_1b_j + \epsilon_{ji}(c_2b'_i - c'_2b_i). \end{aligned} \tag{13}$$

In terms of these we have (in two-vector notation)

$$\begin{aligned} E'^*E + F_j'^*F_j &= (S_1^2 + S_3^2 - \boldsymbol{S}_2^2)N^2 + S_1\boldsymbol{S}_2\times(N'\boldsymbol{L} - N\boldsymbol{L}') + S_3\boldsymbol{S}_2\,.\,(N'\boldsymbol{L} + N\boldsymbol{L}') + 2S_1S_3\boldsymbol{L}\times\boldsymbol{L}' \\ &\quad + 2(\boldsymbol{S}_2\,.\,\boldsymbol{L}')(\boldsymbol{S}_2\,.\,\boldsymbol{L}) + 2(S_1^2 - \tfrac{1}{2}K^2)\boldsymbol{S}_2^2(\boldsymbol{L}\,.\,\boldsymbol{L}'). \end{aligned}$$

We observe that the dependence on the anomalous moment due to the wavefunctions is absorbed in the quantities (13)†.

The polarization sum can now be done by means of the formula

$$\sum_{\text{pol}}(\boldsymbol{X}\,.\,\boldsymbol{\epsilon})(\boldsymbol{Y}\,.\,\boldsymbol{\epsilon}) = \boldsymbol{X}\,.\,\boldsymbol{Y} \tag{14}$$

which holds for any two-vectors $\boldsymbol{X}$, $\boldsymbol{Y}$ of the form $\boldsymbol{X} = \boldsymbol{x} - (x_v/q_v)\boldsymbol{q}$ and is easily proved

† One may show by invariance arguments analogous to those given in a previous paper (Becker 1975), that the particular combinations of the functions (13) appearing in the formula above are the only possible ones.

by geometrical arguments (cf Brown and Kibble 1964). We have already noted that the vector $\boldsymbol{R}$ has the appropriate form (see equation (11)). Performing the sum we obtain

$$\begin{aligned}
Z(u,u') = \frac{m^2q_v^2}{8p_v^2p_v'^2}\Bigg\{&\boldsymbol{R}\cdot\boldsymbol{R}'\left[K^2\left(\frac{p_v+p_v'}{q_v}\right)^2+\left(1+\frac{\mu^2}{2}\right)(S_1^2+S_3^2-S_2^2)\right]\\
&+2(S_1^2-S_3^2)\left[\left(1+\frac{\mu}{2}\right)^2+\frac{\mu^2}{4}[(\boldsymbol{R}\cdot\boldsymbol{R}')^2-(\boldsymbol{R}\times\boldsymbol{R}')^2]\right]+2\mu^2S_1S_3(\boldsymbol{R}\cdot\boldsymbol{R}')(\boldsymbol{R}\times\boldsymbol{R}')\\
&-\mu(1+\tfrac{1}{2}\mu)[2(\boldsymbol{S}_2\cdot\boldsymbol{R})^2+2(\boldsymbol{S}_2\cdot\boldsymbol{R}')^2-S_2^2(\boldsymbol{R}^2+\boldsymbol{R}'^2)]\\
&-(1+\mu)(1+\tfrac{1}{2}\mu)\{S_1\boldsymbol{S}_2\cdot(\boldsymbol{R}-\boldsymbol{R}')-S_3[\boldsymbol{S}_2\times(\boldsymbol{R}+\boldsymbol{R}')]\}\\
&-\tfrac{1}{2}\mu(1+\mu)[\![(\boldsymbol{R}\cdot\boldsymbol{R}')\{S_1\boldsymbol{S}_2\cdot(\boldsymbol{R}-\boldsymbol{R}')-S_3[\boldsymbol{S}_2\times(\boldsymbol{R}+\boldsymbol{R}')]\}\\
&-(\boldsymbol{R}\times\boldsymbol{R}')\{\boldsymbol{S}_1\cdot[\boldsymbol{S}_2\times(\boldsymbol{R}+\boldsymbol{R}')]+S_3\boldsymbol{S}_2\cdot(\boldsymbol{R}-\boldsymbol{R}')\}]\!]\Bigg\}.
\end{aligned} \tag{15}$$

This formula holds for arbitrary polarization of the laser.

4. Transition rate for circular polarization

For the transition rate we have to consider

$$\mathscr{R} = \frac{1}{2V}\sum_{\text{spin}}\sum_{\text{pol}}|\langle p'q\epsilon'|p\rangle|^2 \tag{16}$$

where V is the four-dimensional interaction volume. In this expression we encounter the integral

$$I = \int \mathrm{d}u\,\mathrm{d}u'\,Z(u,u')\exp[\mathrm{i}(\phi(u)-\phi(u'))]. \tag{17}$$

The phase becomes transparent if we use

$$\beta_i(u) = \int^u \mathrm{d}\bar{u}\; a_i(\bar{u}), \qquad u+H(u) = \int^u \mathrm{d}\bar{u}\; a_i(\bar{u})a_i(\bar{u}).$$

Then we have

$$\phi = \omega(\surd 2)(uM+eaP_i\beta_i+\zeta H) \tag{18}$$

where we have introduced the parameters (cf Brown and Kibble 1964)

$$\begin{aligned}
&v = \frac{ea}{m} \qquad \zeta = v^2\zeta_0 \qquad \zeta_0 = \frac{m^2q_v}{(2\surd 2)\omega p_vp_v'}\\
&M = \frac{m^2}{4\zeta_0}\boldsymbol{P}^2+\zeta+\zeta_0 \qquad \boldsymbol{P} = \frac{1}{\omega\surd 2}\left(\frac{\boldsymbol{p}}{p_v}-\frac{\boldsymbol{p}'}{p_v'}\right).
\end{aligned} \tag{19}$$

We shall now concentrate on a circularly polarized laser wave of 'infinite' duration as the limiting case of a beam, which is switched on/off in the remote past/future. In this case we have

$$\beta_1 = \sin(\surd 2)\omega u, \qquad \beta_2 = \cos(\surd 2)\omega u, \qquad H(u) = 0. \tag{20}$$

With

$$P_1 = P\cos\tau, \qquad P_2 = P\sin\tau \tag{21}$$

we obtain an expression for the exponential in equation (17), which can be expanded in terms of Bessel functions

$$\exp[\mathrm{i}(\phi(u)-\phi(u'))] = \sum_{n=-\infty}^{+\infty} \mathrm{J}_n(2eaP\cos\sigma)\exp[\mathrm{i}\Delta(M+\tfrac{1}{2}n)] \tag{22}$$

where we have used the variables

$$\sigma = \tau + \frac{\omega}{\sqrt{2}}(u+u'), \qquad \Delta = (\sqrt{2})\omega(u-u') \tag{23}$$

which can be used as integration variables in equation (17).

The integration interval is infinite for both variables. The expressions for the quantities (13) in the case of circular polarization are periodic functions of σ, as well as the exponential (22). Therefore we may write

$$I = \frac{1}{2\omega^2}\int \mathrm{d}\sigma\,\mathrm{d}\Delta\, Z\,\mathrm{e}^{\mathrm{i}(\phi-\phi')} = \frac{L}{4\pi\omega^2}\int_0^{2\pi}\mathrm{d}\sigma\int_{-\infty}^{+\infty}\mathrm{d}\Delta\, Z\,\mathrm{e}^{\mathrm{i}(\phi-\phi')} \tag{24}$$

Here L is the ('infinite') length of the integration interval. L drops out (as do the divergences contained in the formal appearance of squares of δ functions), if we divide by the interaction volume. The integration on Δ will lead to another δ function, so that we obtain the well known formula for the transition rate

$$\Gamma = (\sqrt{2})\omega\mathscr{R}. \tag{25}$$

It is this latter δ function, which gives rise to the Compton-type formula expressing the scattered frequency ω_q in terms of ω and the electron momentum variables.

In order to compute Γ we have to insert the results for the functions (13) for circular polarization, which have been derived in a previous paper (Becker 1975). These functions contain the anomalous moment in the combinations

$$g = -\tfrac{1}{2}\nu\mu \qquad \text{and} \qquad \rho = \tfrac{1}{2}[1-\sqrt{(1+4g^2)}] = -g^2+3g^4-\ldots \tag{26}$$

which are small quantities even for high laser intensities: for electrons one could (optimistically) expect ν to be at most of order unity, but μ is small; for nucleons ν is small. We shall, however, not expand the functions (13), since we want to obtain the frequency shift exactly. The integrations in equation (24) can be performed analytically. Each term contains a δ function of the type

$$\delta(p'_v+q_v-p_v)\,\delta(p'_i+q_i-p_i)\,\delta(M-r^{(i)}) = (\sqrt{2})\omega\,\delta(p'+q-p-k(r^{(i)}-\zeta))$$

where (i) stands for (0), (+), (−) respectively and we have

$$r^{(0)} = r, \qquad r^{(\pm)} = r\mp 2\rho \tag{27}$$

with integer r. The final result reads

$$\Gamma = \frac{(2\pi)^4 e^2K^2m^2}{2p_vp'_v}\sum_r\sum_{i=(0),(+),(-)}\delta(p'+q-p-k(r^{(i)}-\zeta))B_r^{(i)}. \tag{28}$$

The summation has to be performed on positive values of r in the (0) and (−) term, since otherwise the argument of the δ function cannot be zero as a consequence of $M > 0$.

In the $(+)$ term also $r = 0$ contributes, since ρ is negative. The exact expressions for the coefficients are rather long. In order to write them down in a transparent form we introduce the following abbreviations:

$$A_r = J_{r+1}^2 + J_{r-1}^2 \qquad C_r = J_{r-1}J_{r+1}$$
$$B_r = J_r^2 \qquad D_r^{(\pm)} = \frac{\xi}{2\zeta_0} J_r J_{r\mp 1}. \tag{29}$$

Here the argument of all Bessel functions J_r is the variable

$$\xi = eaP \tag{30}$$

Furthermore we use

$$z^{(i)} = \frac{(r^{(i)} - \zeta - \zeta_0)}{\zeta_0} \equiv z. \tag{31}$$

Then we have

$$\begin{aligned} B_r^{(0)} = {} & \frac{1}{2}\left(1 + \frac{(qk)\zeta_0}{m^2}\right)[v^2 A_r - 2(1+v^2)B_r] + \frac{(qk)\zeta_0}{2m^2(1+4g^2)}\Big[A_r[\tfrac{1}{4}v^2\mu^2 + gv^3\tfrac{1}{2}\mu(1+\mu) \\ & + 2g^2(1+\tfrac{1}{2}\mu)^2 - 2g^2v^2 - 2g^2v^2\mu(1+\tfrac{1}{2}\mu) + g^2v^4\tfrac{1}{2}\mu^2 + g^2\tfrac{1}{2}\mu^2 z^2] \\ & + g\mu z C_r[v(1+\mu) + 2g\mu(v^2-1) - 4g] + B_r\Big(-\tfrac{1}{2}\mu^2(1+v^2) + 4g^2(1+v^2) \\ & + 4g^2\mu(z+1+v^2) - 2g^2\mu^2(z^2-1+v^4) - \frac{g}{v}(1+\mu)(z+1-v^2) \\ & - \frac{g}{2v}\mu(1+\mu)(z^2+2z+2+2v^2+3v^4)\Big)\Big] \end{aligned} \tag{32}$$

$$\begin{aligned} B_r^{(\pm)} = {} & \frac{(qk)\zeta_0}{2m^2(1+4g^2)}\Big\{B_r(1-2\rho)[1+\tfrac{1}{2}\mu(2+gv^3)+\tfrac{1}{4}\mu^2(1+z^2-v^4+2gv^3)] \\ & + A_{r\mp 1}\{(g^2+\rho+g^2v^2)(1+\mu) - \tfrac{1}{2}\mu g v^3(1-\rho) + \tfrac{1}{4}\mu^2(1+z^2)(g^2+\rho) \\ & + \tfrac{1}{4}\mu^2[4g^2v^2 + v^4(1+g^2-\rho) - 2gv^3(1-\rho)]\} + (1-2\rho)D_r^{(\pm)} \\ & \times\left(\frac{g}{v} + \frac{\mu g}{2v}(z+3-3v^2) - \mu^2(z-v^2) + \frac{\mu^2 g}{2v}(z+1-3v^2)\right) + zC_{r\mp 1} \\ & \times\{\tfrac{1}{2}\mu g(4g-v) + \tfrac{1}{2}\mu^2[v^2+2g^2(1+v^2)-vg]\} + B_{r\mp 1}[2g^2 z - vg - \tfrac{1}{2}g\mu v(3+2z) \\ & + \tfrac{1}{2}\mu^2[2z(v^2+g^2+2v^2g^2) - vg(1+2z)] + \frac{(r\mp 1)}{\zeta_0}\Big(\frac{\rho g}{v} - 2g^2 + \frac{g\mu}{2} \\ & \times[3v(1-\rho)-4g] + \frac{g\mu\rho}{2v}(3+z) - \mu^2[2g^2 + z(g^2+\rho) + v^2(1+g^2-\rho)] \\ & + \tfrac{3}{2}\mu^2 vg(1-\rho) + \frac{\mu^2 g\rho}{v}(z+1)\Big)\Big]\Big\} \end{aligned} \tag{33}$$

We have retained the original dependence on g, μ and v in spite of relation (26), since we want to recover the corresponding formulae for the neutron, which has only magnetic interactions. This case is obtained by picking up only the terms of order μ^2 (the linear terms are interference terms between the electric and magnetic part) and putting $\nu = \xi = 0$.

For $\mu = g = 0$ only the first expressions in (32) and (33) contribute and the transition rate reduces to the corresponding one for a particle without anomalous moment, which has been derived earlier (Brown and Kibble 1964). It is evident from equations (32) and (33) that the terms due to the magnetic interaction are relativistic effects, since they are proportional to $(qk)/m^2$. For ordinary laser frequencies this invariant is a very small parameter, so that it will be difficult to measure the magnetic contributions. The effects could perhaps become observable, if it were possible to construct intense lasers with very high frequencies (Baldwin and Khokhlov 1975).

5. Cross section

In computing the cross section it has to be observed that the normalization of the wavefunction and the units used here are such that the particle density in the initial state is

$$\rho_{\text{in}} = \frac{4\pi}{a^2\omega}\frac{p_v\sqrt{2}}{K\omega_p}, \qquad \omega_p = (p^2+m^2)^{1/2}$$

(correspondingly for the final state with $a = 1$). The differential cross section (per laser photon) consists of incoherent contributions corresponding to harmonic generation:

$$\mathrm{d}\sigma = \sum_{r=1}^{\infty} (\mathrm{d}\sigma_r^{(0)} + \mathrm{d}\sigma_r^{(-)}) + \sum_{r=0}^{\infty} \mathrm{d}\sigma_r^{(+)} \tag{34}$$

where we have in the laboratory frame

$$\frac{\mathrm{d}\sigma_r^{(i)}}{\mathrm{d}\Omega} = R_0^2 \left(\frac{\omega_q}{\omega}\right)^2 \frac{2}{\nu^2 r^{(i)}} B_r^{(i)}. \tag{35}$$

Here $R_0 = e^2/m$ is the classical electromagnetic radius of the particle. The variables contained in B read in the laboratory frame in terms of the scattering angle θ

$$z = \cot^2\theta/2, \qquad \xi = 2\nu\zeta_0 \cot\theta/2 = \frac{\nu r^{(i)} \sin\theta}{1+\nu^2\sin^2\theta/2}. \tag{36}$$

The formula for the frequency shift is (Becker 1975)

$$\frac{1}{\omega_q^{(i)}} = \frac{1}{\omega r^{(i)}} + \left(\frac{2}{m} + \frac{\nu^2}{\omega r^{(i)}}\right) \sin^2\theta/2. \tag{37}$$

The last term is the intensity-dependent frequency shift encountered in nonlinear Compton scattering, which has been found for $i = 0$ before (Brown and Kibble 1964). Because of the presence of $r^{(i)}$ the shifted frequencies appear in triplets of very small separation (cf equation (27)). The most interesting feature is the appearance of a very low frequency due to the fact that r can be zero for $(i) = (+)$. Since ρ is very small, this frequency is a radio frequency. At $\theta = 180°$ we obtain for an infrared laser ($\lambda = 1{\cdot}06\ \mu$m) with an intensity factor $\nu^2 = 1$ for electrons a scattered frequency of 95 MHz. For protons and the same laser data the intensity factor is 3×10^{-7} and we have 135 MHz.

6. Discussion

For the radio frequency term $(i) = (+)$, $r = 0$ the argument of all Bessel functions is small for any v (ie both for electrons and nucleons). The leading term in a power series expansion yields

$$\frac{d\sigma_0^{(+)}}{d\Omega} = R_0^2 \frac{v^4\mu^6}{4}\left(\frac{\omega}{m}\right)^2\left(\frac{(1+\frac{1}{2}\mu)^2 \sin^4\theta/2 + \frac{1}{4}\mu^2\cos^4\theta/2}{(1+v^2\sin^2\theta/2)^4} + O(g^2)\right). \tag{38}$$

For the laser data used above the second factor is 6×10^{-10} for electrons and 7×10^{-13} for protons. The third factor amounts to 5×10^{-12} and $1{\cdot}6\times10^{-18}$ respectively. Thus we obtain for the total cross section a value of $1{\cdot}5\times10^{-55}\,\mathrm{cm}^2$ for electrons and $5\times10^{-61}\,\mathrm{cm}^2$ for protons, which leaves little hope for observation. An increase of the laser energy would enlarge the third factor considerably, but it has to be observed that a loss in intensity would easily overcompensate this gain. Probably the only chance is thus to hope for an enhancement by resonance effects due to stimulation, which have been discussed for particles without anomalous moment before (Oleinik 1968).

Next we investigate the scattering off neutrons, which have only magnetic interaction with the laser. Here only four contributions are different from zero:

$$(i) = (0):\ r = 1, \qquad (i) = (+):\ r = 0, 1, 2. \tag{39}$$

The results can be summarized by the formulae

$$\frac{d\sigma_r^{(i)}}{d\Omega} = R_0^2\left(\frac{\omega}{m}\right)^2\frac{\mu^4}{4}F_r^{(i)}, \qquad \frac{1}{\omega_q} = \frac{1}{f_r^{(i)}\omega} + \frac{2}{\kappa}\sin^2\theta/2 \tag{40}$$

where the factors F and f are given by

$$F_1^{(0)} = \frac{1+\cos^2\theta}{2[1+(2\omega/m)\sin^2\theta/2]^3}, \qquad f_1^{(0)} = 1 \tag{40a}$$

$$F_1^{(+)} = \frac{\sin^2\theta}{2[1+(2\omega/m)\sin^2\theta/2]^3}, \qquad f_1^{(+)} = 1+2g^2 \tag{40b}$$

$$F_2^{(+)} = \frac{3}{4}(v\mu)^2\frac{1+\cos^2\theta}{2[1+(4\omega/m)\sin^2\theta/2]^3}, \qquad f_2^{(+)} = 2(1+g^2) \tag{40c}$$

$$F_0^{(+)} = \tfrac{1}{4}(v\mu)^4\tfrac{1}{2}(1+\cos^2\theta), \qquad f_0^{(+)} = 2g^2 \tag{40d}$$

Thus $F_1^{(0)}$ and $F_1^{(+)}$ differ by the angular dependence: $F_1^{(+)}$ vanishes in the forward and backward direction, where $F_1^{(0)}$ is peaked. For $\cos^2\theta = \frac{1}{3}$ we have $F_1^{(0)} = F_1^{(+)}$. Due to the small difference of the corresponding frequencies the emitted radiation should show a very characteristic beat pattern.

The total cross section is of the order of $(\omega/m)^2$ times $\frac{2}{3}\mu b$ or $\frac{1}{3}\mu b$ for the lowest harmonic term (a) or (b) respectively. The second harmonic term (c) is smaller by a factor $3g^2$ and the radio frequency term (d) by a factor $4g^4$.

The particular pattern, that only the four given amplitudes are different from zero, is specific for circular polarization. In order to understand the pattern, we shall look at the quasi-levels of a neutron in a circularly polarized wave field, which are indicated in figure 1. The level scheme may be obtained from the Fourier transform $\tilde{\Psi}_p(p')$ $(p^2 = m^2)$ of the neutron wavefunction, which is a sum of terms proportional to

$$\delta(p' - p \pm \tfrac{1}{2}k[1 \pm (1+4g^2)^{1/2}]).$$

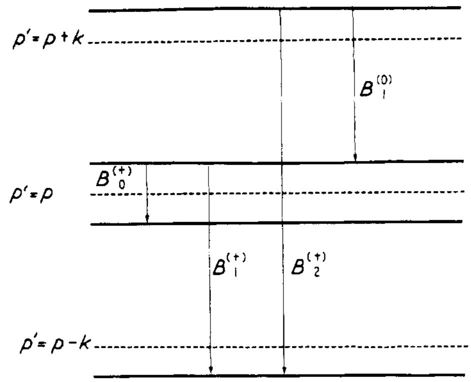

Figure 1. Level scheme of a neutron in a circularly polarized laser field. Full lines: displaced levels; broken lines: undisplaced levels.

The structure obtained is due to the definite helicity of the laser quanta and the fact that the magnetic interaction $\sigma_{\mu\nu}F^{\mu\nu}$ connects only states of opposite helicity; because different spin precession components with opposite helicity are displaced in opposite directions on the energy scale. The interaction with the non-laser photon induces the transitions indicated in figure 1 in accordance with (39).

For charged particles the formulae remain complicated. For electrons the magnetic terms are, due to the small anomalous moment, small corrections to the relativistic contribution from the Dirac current. For protons the magnetic terms are of the same order of magnitude, but all relativistic effects are very small in this case. In general there is a remarkable asymmetry between the (+) and (−) terms, since $B_r^{(+)} \neq B_r^{(-)}$. The relative difference depends on the scattering angle and is of the order of μ. It can thus be neglected for electrons, but is important for nucleons.

Note added in proof. In radio astronomy signals of the order of $10^{-29}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$ are registered. Receivers of this sensitivity should be able to detect the radio frequency term (38) in principle. A careful analysis of the experimental situation seems desirable.

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